Is the doubler of the electron an antiquark?

Michael Creutz [a] *

[a]Physics Department, Brookhaven National Laboratory, PO Box 5000, Upton, NY 11973-5000, USA
creutz@wind.phy.bnl.gov

Reassigning degrees of freedom in the standard model allows an interpretation where the SU(2) gauge group is vector-like and parity violation moves to the strong and electromagnetic interactions. In this picture, the electron is paired with an anti-quark. Requiring exact gauge invariance for the electromagnetic interaction clarifies the mechanism behind a recent proposal for a lattice regularization of the standard model.

The standard model of elementary particle interactions is based on the product of three gauge groups, $SU(3)\times SU(2)\times U(1)_{em}$. Here the $SU(3)$ represents the strong interactions of quarks and gluons, the $U(1)_{em}$ corresponds to electromagnetism, and the $SU(2)$ gives rise to the weak interactions. I ignore here the technical details of electroweak mixing.

The full model is, of course, parity violating, as necessary to describe observed helicities in beta decay. This violation is normally considered to lie in the $SU(2)$ of the weak interactions, with both the $SU(3)$ and $U(1)_{em}$ being parity conserving. However, this is actually a convention, adopted primarily because the weak interactions are small. I argue below that reassigning degrees of freedom allows a reinterpretation where the $SU(2)$ gauge interaction is vector-like. Since the full model is parity violating, this process shifts the parity violation into the strong, electromagnetic, and Higgs interactions. The resulting theory pairs the left handed electron with a right handed anti-quark to form a Dirac fermion.

With a vector-like weak interaction, the chiral issues which complicate lattice formulations now move to the other gauge groups. Requiring gauge invariance for the re-expressed electromagnetism then clarifies the mechanism behind a recent proposal for a lattice regularization of the standard model.

To begin, consider only the first generation, which involves four left handed doublets. These correspond to the neutrino/electron lepton pair plus three colors for the up/down quarks

$$\begin{pmatrix} \nu \\ e^- \end{pmatrix}_L, \begin{pmatrix} u^r \\ d^r \end{pmatrix}_L, \begin{pmatrix} u^g \\ d^g \end{pmatrix}_L, \begin{pmatrix} u^b \\ d^b \end{pmatrix}_L \tag{1}$$

Here the superscripts from the set $\{r, g, b\}$ represent the internal $SU(3)$ index of the strong interactions, and the subscript L indicates left-handed helicities.

If I ignore the strong and electromagnetic interactions, leaving only the weak $SU(2)$, each of these four doublets is equivalent and independent. I now arbitrarily pick two of them and do a charge conjugation operation, thus switching to their antiparticles

$$\begin{matrix} \begin{pmatrix} u^g \\ d^g \end{pmatrix}_L \longrightarrow \begin{pmatrix} \overline{d^g} \\ \overline{u^g} \end{pmatrix}_R \\ \begin{pmatrix} u^b \\ d^b \end{pmatrix}_L \longrightarrow \begin{pmatrix} \overline{d^b} \\ \overline{u^b} \end{pmatrix}_R \end{matrix} \tag{2}$$

In four dimensions anti-fermions have the opposite helicity; so, I label these new doublets with R representing right handedness.

With two left and two right handed doublets, I now combine them into two Dirac doublets

$$\begin{pmatrix} \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \\ \begin{pmatrix} \overline{d^g} \\ \overline{u^g} \end{pmatrix}_R \end{pmatrix} \qquad \begin{pmatrix} \begin{pmatrix} u^r \\ d^r \end{pmatrix}_L \\ \begin{pmatrix} \overline{d^b} \\ \overline{u^b} \end{pmatrix}_R \end{pmatrix} \tag{3}$$

*This manuscript has been authored under contract number DE-AC02-76CH00016 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

Formally in terms of the underlying fields, the construction takes

$$\begin{array}{l}\psi = \frac{1}{2}(1-\gamma_5)\psi_{(\nu,e^-)} + \frac{1}{2}(1+\gamma_5)\psi_{(\overline{d^g},\overline{u^g})} \\ \chi = \frac{1}{2}(1-\gamma_5)\psi_{(u^r,d^r)} + \frac{1}{2}(1+\gamma_5)\psi_{(\overline{d^b},\overline{u^b})}\end{array} \qquad (4)$$

From the conventional point of view these fields have rather peculiar quantum numbers. For example, the left and right parts have different electric charges. Electromagnetism now violates parity. The left and right parts also have different strong quantum numbers; the strong interactions violate parity as well. Finally, the components have different masses; parity is violated in the Higgs mechanism.

The different helicities of these fields also have variant baryon number. This is directly related to the known baryon violating processes through weak "instantons" and axial anomalies[1]. When a topologically non-trivial weak field is present, the axial anomaly arises from a level flow out of the Dirac sea [2]. This generates a spin flip in the fields, *i.e.* $e^-_L \to (\overline{u^g})_R$. Because of the peculiar particle identification, this process does not conserve charge, with $\Delta Q = -\frac{2}{3}+1 = \frac{1}{3}$. This would be a disaster for electromagnetism were it not for the fact that simultaneously the other Dirac doublet also flips ${d^r}_L \to (\overline{u^b})_R$ with a compensating $\Delta Q = -\frac{1}{3}$. This is anomaly cancelation, with the total $\Delta Q = \frac{1}{3} - \frac{1}{3} = 0$. Only when both doublets are considered together is the $U(1)$ symmetry restored. In this anomalous process baryon number is violated, with $L + Q \to \overline{Q} + \overline{Q}$. This is the famous " 't Hooft vertex" [1].

So far the discussion has been in the continuum. Now I turn to the lattice, and use the Kaplan-Shamir approach for fermions [3]. In this picture, our four dimensional world is a "4-brane" embedded in 5-dimensions. The complete lattice is a five dimensional box with open boundaries, and the parameters are chosen so the physical quarks and leptons appear as surface zero modes. The elegance of this scheme lies in the natural chirality of these modes as the size of the extra dimension grows.

With a finite fifth dimension a doubling phenomenon remains, coming from interfaces appearing as surface/anti-surface pairs. It is natural to couple a four dimensional gauge field equally to both surfaces, giving rise to a vector-like theory.

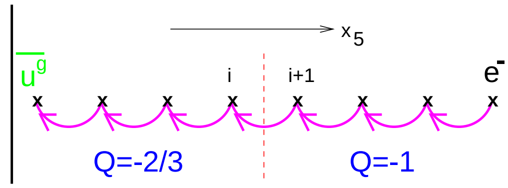

Figure 1. Pairing the electron with the anti-green-up-quark.

I now insert the above pairing into this five dimensional scheme. In particular, I consider the left handed electron as a zero mode on one wall and the right handed anti-green-up-quark as the partner zero mode on the other wall, as sketched in Fig. 1. This provides a lattice regularization for the $SU(2)$ of the weak interactions.

However, since these two particles have different electric charge, $U(1)_{EM}$ must be broken in the interior of the extra dimension. I now proceed in analogy to the "waveguide" picture[4] and restrict this charge violation to ΔQ to one layer at some interior $x_5 = i$. Then the fermion hopping term from $x_5 = i$ to $i+1$

$$\overline{\psi}_i P \psi_{i+1} \qquad (P = \gamma_5 + r) \qquad (5)$$

is a $Q = 1/3$ operator. At this layer, electric charge is not conserved. This is unacceptable and needs to be fixed.

To restore the $U(1)$ symmetry one must transfer the charge from ψ to the compensating doublet χ. For this I replace the sum of hoppings with a product on the offending layer

$$\overline{\psi}_i P \psi_{i+1} + \overline{\chi}_i P \chi_{i+1} \longrightarrow \overline{\psi}_i P \psi_{i+1} \times \overline{\chi}_i P \chi_{i+1} \qquad (6)$$

This introduces an electrically neutral four fermi operator. Note that it is baryon violating, involving a "lepto-quark/diquark" exchange, as sketched in Fig. 2. One might think of the operator as representing a "filter" at $x_5 = i$ through which only charge compensating pairs of fermions can pass.

In five dimensions there is no chiral symmetry. Even for the free theory, combinations like

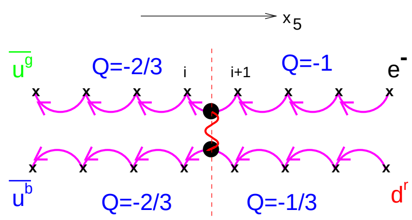

Figure 2. Transferring charge between the doublets.

$\overline{\psi}_i P\psi_{i+1}$ have vacuum expectation values. I use such as a "tadpole," with χ generating an effective hopping for ψ and *vice versa.*

Actually the above four fermion operator is not quite sufficient for all chiral anomalies, which can also involve right handed singlet fermions. To correct this I need explicitly include the right handed sector, adding similar four fermion couplings (also electrically neutral).

Having fixed the $U(1)$ of electromagnetism, I restore the strong $SU(3)$ with an antisymmetrization $Q^rQ^gQ^b \longrightarrow \epsilon^{\alpha\beta\gamma}Q^\alpha Q^\beta Q^\gamma$. Note that similar left-right inter-sector couplings are needed to correctly obtain the effects of topologically nontrivial strong gauge fields.

For the strong interactions alone I could replace the four fermion vertices with simple hoppings, matching particles with their usual partners. This leads to the usual chiral predictions, such as $M_Q \sim m_\pi^2$ in $L_5 \to \infty$ limit, and forms the basis of the formulation of Ref. [5]. Recent tests are encouraging that this may be numerically advantageous [6].

An alternative view is to fold the lattice about the interior of the fifth dimension, placing all light modes on one wall and having the multi-fermion operator on the other. This is the model of Ref. [7], with the additional inter-sector couplings correcting a technical error [8].

Unfortunately the scheme is still non rigorous. In particular, the non-trivial four fermion coupling represents a new defect and we need to show that this does not give rise to unwanted extra zero modes. Note, however, that the five dimensional mass is the same on both sides of defect, removing topological reasons for such.

A second worry is that the four fermion coupling might induce an unwanted spontaneous symmetry breaking of one of the gauge symmetries. We need a strongly coupled paramagnetic phase without spontaneous symmetry breaking. Ref. [7] showed that strongly coupled zero modes preserved the desired symmetries, but the analysis ignored contributions from heavy modes in the fifth dimension.

Assuming all works as desired, the model raises several other interesting questions. As formulated, I used a right handed neutrino to provide all quarks with partners. Is there some variation that avoids this particle, which completely decouples in the continuum limit? Another question concerns possible numerical simulations; is the effective action positive? Finally, we have used the details of the usual standard model, leaving open the question of whether this model is somehow special. Can we always use multi-fermion couplings to eliminate undesired modes in other anomaly free chiral theories? There is much more to do!

REFERENCES

1. G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D14 (1976) 3432.
2. J. Ambjorn, J. Greensite, and C. Peterson, Nucl. Phys. B221 (1983) 381; B. Holstein, Am. J. Phys. 61 (1993) 142.
3. D. Kaplan, Phys. Lett. B288 (1992) 342; M. Golterman, K. Jansen, D. Kaplan, Phys. Lett. B301 (1993) 219; Y. Shamir, Nucl. Phys. B406 (1993) 90; M. Creutz and I. Horvath, Phys. Rev. D50 (1994) 2297.
4. M. Golterman, K. Jansen, D. Petcher, and J. Vink, Phys. Rev. D49 (1994) 1606.
5. V. Furman and Y. Shamir, Nucl. Phys. B439 (1995) 54.
6. T. Blum and A. Soni, Phys. Rev. D56 (1997) 174; hep-lat/9706023.
7. M. Creutz, M. Tytgat, C. Rebbi, S.-S.Xue, Phys. Lett. B402, (1997) 341.
8. H. Neuberger, hep-lat/9705022.